On the stability of large-scale streaks in turbulent Couette and Poiseuille flows

Sur la stabilité des streaks à grande échelle dans les écoulements de Couette et Poiseuille turbulents

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Abstract

The linear secondary stability of large-scale optimal streaks in turbulent Couette flow at $Re_\tau = 52$ and Poiseuille flow at $Re_\tau = 300$ is investigated. The streaks are computed by solving the nonlinear two-dimensional Reynolds-averaged Navier–Stokes equations using an eddy-viscosity model. Optimal initial conditions leading the largest linear transient growth are used, and as the amplitude of the initial vortices increases, the amplitude of streaks gradually increases. Instabilities of the streaks appear when their amplitude exceeds approximately 18\% of the velocity difference between walls in turbulent Couette flow and 21\% of the centerline velocity in turbulent Poiseuille flow. When the amplitude of the streaks is sufficiently large, the instabilities attain significant growth rates in a finite range of streamwise wavenumbers that shows good agreement with the typical streamwise wavenumbers of the large-scale motions in the outer region.

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1. Introduction

Understanding the dynamics of coherent motions is a central issue in the research on wall-bounded turbulent flows. In the near-wall region, streaks, i.e. spanwise alternating patterns of high- and low-momentum regions with mean spacing about one hundred wall units, have been found as the most prominent feature [1]. These streaks sustain independently of the turbulent motions in the outer region [2], and the corresponding process has been understood as a cycle involving amplification of streaks by vortices, breakdown of the streaks via instability and the subsequent nonlinear process generating new vortices [3,4]. Streaky motions, however, have been also found in the outer region, and they carry a significant fraction of turbulent kinetic energy and Reynolds stress [5,6]. The origin of these large-scale streaky motions is not clear yet, but nonmodal stability theory has predicted that they can be significantly amplified by lift-up mechanism [7–9].

This encouraging result suggests that a self-sustaining process similar to the buffer-layer cycle presumably exists in the outer region, as confirmed by recent results [10]. However, streaks maximally amplified are found to be uniform in the streamwise direction, whereas the streaky motions observed in experiments have the finite streamwise wavelengths and meander with vortex packets (also called large-scale motions) coherently aligned to them [5,6].

Currently, there is no sound explanation for these features, and only recently it has been conjectured that the vortex packets may be related to the instability of large-scale streaks [5]. The goal of the present study is to analyze the stability of large-scale streaks and to seek a relationship between the streamwise wavelengths of the instability and the length-scales of the coherent structures in the outer region. In order to theoretically track this issue, we consider an eddy-viscosity model for the organized waves [11] successfully used to study the transient growth of large-scale streaks [7–9], and conduct a secondary stability analysis of the most amplified streaks in the turbulent Couette flow and in the turbulent Poiseulle flow.

2. Background

We consider the turbulent flow of an incompressible fluid with the density $\rho$ and kinematic viscosity $\nu$ in a channel with walls located at ±h. Here, the streamwise, wall-normal and spanwise directions are denoted as $x$, $y$ and $z$ respectively. For Couette flow, the upper and the lower walls move in opposite directions with the same velocity $\frac{\partial U}{\partial t}$, and the solutions are assumed uniform in the streamwise direction. Once the streaky base flow is defined as $u_i(x, y, z, t)$ is computed, the secondary base flow is defined as $U_b(x, y, z) \equiv U(y, z) + u_i(x, y, z)$. The stability of $U_b(x, y, z)$ is then studied by linearizing (1) with the secondary perturbations $u'_i(x, y, z, t)$.

Under the assumption that the base flow $U_b(x, y, z)$ is periodic in the spanwise direction, the Floquet theory allows the linearized equation to have the following normal-mode solution:

$$u'_i(x, y, z, t) = e^{i\alpha x - i\omega t} \sum_{n=-N}^{\infty} \hat{u}'_i(y)e^{i(n+\epsilon)\beta_0 z} + c.c. \quad (2)$$

where $\alpha$ is the streamwise wavenumber, $\omega$ the complex frequency, $\beta_0$ the spanwise wavenumber related to the fundamental period of the base flow, and $0 < \epsilon < 1/2$ is the detuning parameter. According to the values of $\epsilon$, the solutions (2) are classified into ‘fundamental’ ($\epsilon = 0$) and ‘subharmonic’ ($\epsilon = 1/2$) modes. Also, the modes with even and odd symmetries about base flows are called ‘sinuous’ and ‘varicose’ respectively. Further details on the classification of the modes, the readers are referred to [12]. In the present study, we focus only on the fundamental sinuous mode, found as the most unstable one for all the cases considered.

Eq. (1) is discretized using Chebyshev polynomials and Fourier series in the wall-normal and spanwise directions respectively. The time integration used to compute $U_b(y, z)$ is conducted using the Runge–Kutta third-order method. For the Floquet analysis, the same spatial discretization is applied to the linear operators. The resulting numerical eigenvalue problem is then solved using the implicitly restarted Arnoldi method (for further details, see [12]). All the computations here are carried out with $N_y \times N_z = 65 \times 32$.

3. Results

We consider a turbulent Couette flow at $Re_t = 52$ and a Poiseulle flow at $Re_e = 300$. The computation of the streaky base flows is carried out by using the optimal initial conditions, that consist of pairs of the counter-rotating streamwise vortices computed in $[8,9]$ (see also Fig. 2). The spanwise spacing is chosen as $\lambda_z = 4h$ ($\beta_0 h = \pi/2$), which is very near the optimal value $[8,9]$. The spanwise size of computational box is set to as $L_z = \lambda_z$, so that a single pair of optimal initial
at a interval of the streamwise wavenumbers is unstable with maximum growth at the w velocity is the same as the phase speed increasing $Av$. initial vortices and streaks at $t$ tendency generally becomes stronger for larger streak amplitudes. the growth rate gradually increases. Then instability begins to appear ($Av$ range of Poiseulle flows are stable for sufficiently low amplitudes of the streaks ($Av$). Fig. 3 shows the growth rate of fundamental sinuous mode with the streamwise wavenumber $\alpha$ for streaky base flows corresponding to • in Fig. 1: (a) Couette flow with $Av = 0.08, 0.11, 0.12, 0.16$; (b) Poiseulle flow with $Av = 0.06, 0.078, 0.09, 0.1$. Here, ● denotes the maximum of $Av$ at $t = t_{\text{max}}$. Both Couette and Poiseulle flows exhibit large transient amplifications of the streaks through the coherent lift-up effect.

As $\alpha$ increases, the amplitude of the streaks also increases. However, the ratio of the amplification slightly decays with $\alpha$, and the time for $Av$ to reach its maximum ($t_{\text{max}}$) also becomes shorter. Typical cross-stream views of the initial vortices and streaks at $t = t_{\text{max}}$ are reported in Fig. 2. It is seen that the low-momentum regions where the fluid is ejected from the wall by the vortices ($y > 0, z \simeq \pm 2h$) and ($y < 0, z \simeq 0$) in Fig. 2) are narrow and intense, and this tendency generally becomes stronger for larger streak amplitudes.

The Floquet stability analysis is performed for the fundamental modes using the streaky base flows extracted at $t = t_{\text{max}}$. Fig. 3 shows the temporal evolution of the streak amplitude for several amplitudes $Av$ of the optimal initial vortices. Both Couette and Poiseulle flows exhibit large transient amplifications of the streaks through the coherent lift-up effect. As $Av$ increases, the amplitude of the streaks also increases. However, the ratio of the amplification slightly decays with increasing $Av$, and the time for $Av$ to reach its maximum ($t_{\text{max}}$) also becomes shorter. Typical cross-stream views of the initial vortices and streaks at $t = t_{\text{max}}$ are reported in Fig. 2. It is seen that the low-momentum regions where the fluid is ejected from the wall by the vortices ($y > 0, z \simeq \pm 2h$) and ($y < 0, z \simeq 0$) in Fig. 2) are narrow and intense, and this tendency generally becomes stronger for larger streak amplitudes.

The Floquet stability analysis is performed for the fundamental modes using the streaky base flows extracted at $t = t_{\text{max}}$. Fig. 3 shows the growth rate $\omega h$ of sinuous modes versus the streamwise wavenumber $\alpha$ for several $Av$. Both Couette and Poiseulle flows are stable for sufficiently low amplitudes of the streaks ($\omega h < 0$). As the amplitude of streak $Av$ increases, the growth rate gradually increases. Then instability begins to appear ($\omega h > 0$) when $Av$ reaches the value: $Av_{c} = 18\%$ at $\alpha h = 0.3$ for Couette flow (Fig. 3a) and $Av_{c} = 21\%$ at $\alpha h = 1.2$ for Poiseulle flow (Fig. 3b). When $Av > Av_{c}$, a finite interval of the streamwise wavenumbers is unstable with maximum growth at $\omega h \simeq 0.6$ in Couette flow (Fig. 3a) and $\alpha h \simeq 1.4$ in Poiseulle flow (Fig. 3b). We have verified that varicose modes are less unstable than sinuous modes in this range of $Av > Av_{c}$.

Fig. 4 shows the spanwise velocity component $w$ of the streak-instability eigenfunction and the line where the mean velocity is the same as the phase speed $c_{r}$ of instability wave ($c_{r} = 0$ in Couette flow while $c_{r} = 0.87U_{\text{ref}}$ in Poiseulle flow). The $w$-component of the eigenfunction is concentrated in the lifted low-momentum region for both Couette and Poiseulle flows.
Fig. 4. Cross-stream (y-z) view of the absolute value of the spanwise velocity eigenfunction corresponding to in Fig. 3: (a) Couette flow; (b) Poiseulle flow. Here, the contour is normalized by its maximum and the thick dashed line denotes the velocity of streaky base flow which is the same with the phase speed of the streak instability.

4. Discussion

We have investigated the instability of finite amplitude streaks in the framework of an eddy viscosity of turbulent Couette and Poiseulle flows. The nature of the streak instability we find bears some similarity to the one observed in the laminar flows: the instability appears for the high amplitude streaks and it is dominated by the fundamental sinuous mode. An interesting point is that the critical streak amplitudes in the present model are lower than those in laminar flows: for example, our own computations in laminar Couette and Poiseulle flows at Re_h = 500 give $A_{h,c} \approx 23\%$ and $A_{h,c} \approx 28\%$ respectively (see also [13]), whereas for the turbulent Couette and Poiseulle flows it is found $A_{h,c} \approx 18\%$ and $A_{h,c} \approx 21\%$ respectively at $Re_T = 52$ and $Re_T = 300$. However, this does not mean that the streak instability in turbulent flows can be more easily triggered than in laminar flows because as the optimal turbulent transient growth is smaller than the laminar one [8,9], initial vortices in turbulent flows should be more energetic to drive streak instability than those in laminar flows. For example, in the laminar Poiseulle flow, $A_{h,c} \sim O(10^{-3})$ is required to trigger streak instability [13], but the amplitude of initial vortices in the present model of Poiseulle flow is almost two order of magnitude larger (i.e. $A_{h,c} \approx 0.08$).

In the outer region of the turbulent Poiseulle flow, the streamwise length scale of the large-scale motions has been associated with the peak near the high wavenumber boundary of the $\alpha^{-1}$ regime in the spectra of the outer region at $ah = 1 \sim 2$ ($\lambda_h \sim 3.1 \sim 6.3h$), and that is clearly visible for $0 < |y| < 0.6h$ [15,16]. The range of unstable streamwise wavenumbers in the present study is also obtained at $ah = 0.8 \sim 1.8$ ($\lambda_h \sim 3.5 \sim 7.9h$; see Fig. 3b), showing good agreement with the location of the peak in the streamwise spectra. Moreover, the eigenfunctions obtained here have significant values in $0 < |y| < 0.6h$, also comparable to the spectra. For the turbulent Couette flow, the streamwise wavenumber having the maximum growth rate ($ah \sim 0.6$; $\lambda_h \sim 10.5h$; see Fig. 3a) also corresponds well to the location of the peak in the streamwise spectra at the channel center ($ah \sim 0.63$; $\lambda_h \sim 10h$) [17]. This good correlation in the length-scale comparison implies that the large-scale motion could be formed by instability of much longer streaky motions in the outer region. However, as the Reynolds numbers considered here are fairly low, this conclusion remains to be confirmed at larger $Re$. Also, alternative scenario based on the secondary transient growth as in [18] may also be relevant, and is currently under active investigation.

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